Mathematical Modelling of the Drawing of Spun Capillary Tubes

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Summary. We describe a fluid mechanics model that has been constructed in order to allow anunderstanding of the drawing of microstructured optical fibres, or 'holey fibres', to be gained, and furtherour ability to predict and control the final fibre geometry. The effects of fibre rotation are included in the model. Predictions are made by solving the final model numerically.

1 Introduction

Holey fibres consist of a lattice of air holes surrounding a solid core, and are made by heating a macroscopic preform containing holes, and drawing it into fibre form. Such fibres guide light by making use of the effective refractive index difference between two regions of the fibre: the lattice of air holes around the core acts to lower the refractive index in the region surrounding the core. Although at first sight this guidance mechanism is similar to the way in which light is guided in a conventional optical fibre, holey fibres exhibit a host of highly unusual and tailorable optical properties that can often be exploited.

Optical fibres are manufactured by heating a macroscopic preform (a few centimetres in diameter), and drawing it down to the required dimensions (typically 125 μm). However, holey fibre fabrication is often sensitive to the conditions under which the fibre is drawn, and the drawing regimes are more limited than for conventional fibres. A quantitative understanding of the relative effect of changes in one or more of these parameters is required, in order to be able to tailor the geometry of the fibre from a single given preform, by varying these parameters. Ultimately, experimentalists would like to be able to predict more fully the final fibre geometry.

Asymmetry or stress in the fibre profile leads to fibre birefringence, which can have a pronounced impact on holey fibre performance. By introducing a twist into the fibre during the drawing process, the effects of birefringence can be reduced by averaging out the effects of asymmetry, along the length, as is often desirable The periodicity of the twist required depends upon the wavelength of light and the details of the fibre profile In practice, this may be achieved by rotating holey fibre preforms as they enter the furnace, and holding them at zero rotation as they leave the furnace. This leaves the fibre with an overall twist along its length, as required

We show here that the act of imparting a non-zero angular momentum to the fibre as it passes through the furnace can have a significant effect on fibre geometry. We ask at what point this occurs, and if fibre rotation can be used as an additional control parameter in determining the final fibre geometry.

2 Mathematical Modelling

To develop a mathematical model for the process of holey fibre drawing, we begin by considering a single capillary tube. We regard this as a first step towards modelling the general holey fibre problem. To develop a model for capillary drawing that is capable of including the effects of internal hole pressurisation, surface tension and so on, as well as rotation, we begin with the Navier–Stokes and convection–diffusion equations in cylindrical coordinates, and largely follow the methodology set out in [Fitt02]

We assume that the flow is axisymmetric, and therefore independent of the azimuthal angle θ. The velocity $\boldsymbol{q}$ of the molten glass is denoted by $\boldsymbol{q} = w\boldsymbol{e}_z + u\boldsymbol{e}_r + v\boldsymbol{e}_\theta$ where $\boldsymbol{e}_z$, $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ are unit vectors in the z, r and θ directions, respectively, and $v \neq 0$ when rotation is present. A schematic diagram of the capillary geometry is shown in Fig 1

Space permits only the briefest details of the model derivation. After appropriate non-dimensionalisation, an asymptotic analysis of the governing equations, where the ratio of the fibre radius to the fibre length is the key small

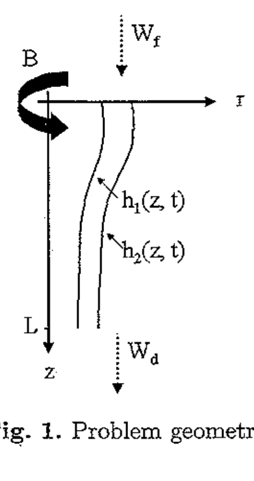

Fig. 1. Problem geometry

parameter, leads, after much manipulation, to the (dimensional) isothermal equations

$$\rho(h_2^2-h_1^2)(w_{0t}+w_0w_{0z}-g)=\Big[3\mu(h_2^2-h_1^2)w_{0z}+\gamma(h_1+h_2)+\frac{\rho}{4}(h_2^4-h_1^4)B^2\Big]_z, \quad (1)$$

$$(h_1^2)_t+(h_1^2w_0)_z=\frac{2p_0h_1^2h_2^2-2\gamma h_1h_2(h_1+h_2)+\rho h_1^2h_2^2B^2(h_2^2-h_1^2)}{2\mu(h_2^2-h_1^2)}, \quad (2)$$

$$(h_2^2)_t+(h_2^2w_0)_z=\frac{2p_0h_1^2h_2^2-2\gamma h_1h_2(h_1+h_2)+\rho h_1^2h_2^2B^2(h_2^2-h_1^2)}{2\mu(h_2^2-h_1^2)}, \quad (3)$$

$$\mu\left((h_2^4-h_1^4)B_z\right)_z=\rho\Big[h_2^2(h_2^2B)_t-h_1^2(h_1^2B)_t\Big]+\rho w_0\Big[h_2^2(h_2^2B)_z-h_1^2(h_1^2B)_z\Big]$$
$$-\frac{\rho\gamma B}{\mu}\left(h_1^2h_2+h_2^2h_1\right)+\frac{\rho^2B^3}{2\mu}\left(h_1^2h_2^4-h_2^2h_1^4\right)+\frac{\rho}{\mu}p_0Bh_1^2h_2^2. \quad (4)$$

Here, density, dynamic viscosity, gravity, surface tension and hole overpressure (i.e. the excess over atmospheric) are denoted by ρ, μ, g, γ and p_0 respectively The inner and outer capillary radii are denoted by h_1 and h_2, w_0 denotes the leading order term in w, and B denotes the variable v_0/r, which may be thought of as an angular frequency Subscripts denote differentiation, t denotes time and z measures the distance along the axis of the capillary. When the fibre rotation is zero, the equations reduce to those derived in [Fitt02]

3 The Effect of Fibre Rotation

Though the practice of spinning solid optical fibres is well established, the spinning of holey fibres has not previously been reported. Evidently, it would be valuable to know whether or not it is possible to spin holey fibres at a rate necessary to obtain the required twist periodicity along the length of the fibre, whilst still retaining the prescribed geometry required by the experimenter, and preventing possible hole collapse

As a model validity check, we first examine at what rate of rotation the geometry of a solid fibre ($h_1 = 0$) begins to be influenced by rotation. When (1)–(4) are presented in non-dimensional form, we note immediately that the rotation equation, (4), decouples from the momentum equation, (1), when the rotation is small. For steady-state fibre drawing, ignoring the complications of inertial forces, surface tension, hole pressurisation and gravity, we find that

$$3\bar{\mu}(\bar{h}_2^2\bar{w}_{0\bar{z}})_{\bar{z}}=-\frac{1}{4}ReS^2(\bar{h}_2^4\bar{B}^2)_{\bar{z}}, \quad (5)$$

where $S=\frac{\Omega L}{W}$, $Re=\frac{LW\rho}{\mu_0}$, Ω and W are a typical angular frequency and downstream velocity respectively, and overbars denote non-dimensional variables. Rotation therefore first begins to significantly influence fibre geometry when $3\bar{\mu} \sim \frac{ReS^2}{4}$. Using typical parameter values shows that this occurs when $\Omega \approx 40\,\mathrm{rad/s}$, a conclusion that may be corroborated by solving (1)–(4) numerically. This is consistent with the rate at which fibres are spun experimentally [Wai]. Although the point at which holey fibre geometry is modified may be somewhat different, (5) provides a useful first approximation

3.1 Numerical Results

A great many asymptotic limits of the equations (1)–(4) may be considered. We do not examine any of these here, referring the reader instead to [Fitt] Instead, we briefly consider the results of some numerical studies carried out on the steady version of the equations, using standard library routines to solve the boundary value problem.

Figure 2 shows the effects of fibre rotation on both a thin- and a thick-walled capillary. We assumed that the glass involved was Suprasil F300, commonly used in the production of low-loss optical fibres The physical properties used for the computations were taken from [Fitt01]. Of course, both the model and the method are applicable to general fluids.

The results in Fig 2 demonstrate that rotation causes the outer fibre radii to increase, predominantly at the top of the furnace. It may be confirmed that the general effect of rotating the preform as it enters the furnace is to increase both the inner and outer radii of the fibre along the entirety of the draw length Fibre rotation may thus be used as an additional control in the drawing process, since it is the fibre dimensions at the end of the furnace that

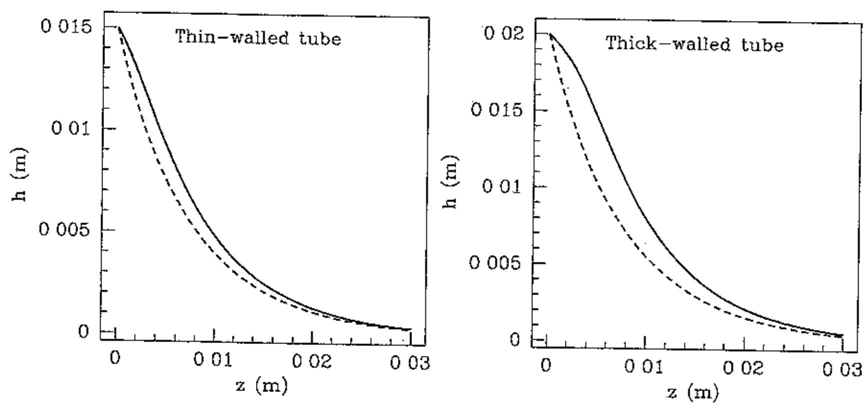

Fig. 2. The effects of fibre rotation on outer capillary radius. (Draw length $L = 0.03\,\mathrm{m}$, temperature $T = 2200\,\mathrm{C}$, draw speed $W_d = 25\,\mathrm{m/min}$, feed speed $W_f = 15\,\mathrm{mm/min}$, rotation rate $\Omega = 35\,\mathrm{rad/s}$.) Each diagram shows the quantity h_2 for fibre pulls with and without rotation (*left diagram*: thin-walled tube ($h_1(0) = 0\,01\,\mathrm{m}$, $h_2(0) = 0\,015\,\mathrm{m}$), *right diagram*: thick-walled tube ($h_1(0) = 0\,01\,\mathrm{m}$, $h_2(0) = 0\,02\,\mathrm{m}$) In both cases, the *lower* of the two curves is the case with no rotation The *upper curves* both show a 'bulge' resulting from the effect of fibre rotation.

primarily concern us. We also note that rotation appears to act on the fibre in a way that counteracts the effects of surface tension, which otherwise tends to close the air holes in the fibre. As well as reducing birefringence, rotation may thus, for example, allow fibres to be drawn at increased temperatures; this is advantageous from a manufacturing point of view as fibres drawn at high temperatures often possess superior strength.

It is also clear from Fig. 2 that the thick-walled tube experiences a much greater deformation than the thin-; this is largely because the initial outer radius of the thick-walled capillary is larger than that of the thin-walled capillary. To gain more insight into the results, we have compared the magnitudes of respective changes in fibre radii as a result of spinning the different fibre types. This reveals that, for both thin- and thick-walled tubes, the inner radius increases more than the outer. The fluid near to the outer edges of the fibre rotates faster and therefore experiences more of an effect due to the rotation than the fluid near to the central hole. The displacement of the outer edge of the fibre, coupled with mass conservation requirements, requires that the inner portions of the fibre must undergo larger changes in radial position.

4 Conclusions

An asymptotic model was constructed, and solved numerically to determine the steady state final fibre geometry. The model provides an accurate way to predict experimental draw results for the case of capillary tubes. Ultimately, it is holey fibres with an arbitrary cross-section that are of interest to us. The model outlined above provides a starting point for holey fibre modelling.

Of more immediate significance, it suggests that the rotation of holey fibres should be possible, in so much as it is possible to rotate them rapidly without causing catastrophic geometrical effects such as fibre explosion, and whilst directly preventing others, such as surface tension pinch-off. Experimental results will soon be obtained to allow us to test these predictions.

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