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On the Separation of Coconuts: A Modeling Week Study*

Alistair D. Fitt†
Colin P. Please†

Abstract. The modus operandi of "modeling weeks" is explained and illustrated using a classical modeling week problem of subjecting coconuts to an external pressure before splitting them open, in the hope that the coconut meat will then separate easily from the shell. A range of modeling approaches is considered, and a poroelastic model is posed that allows the key problem parameters to be identified and calculated. We conclude that separation by this means is likely to be possible and relatively easy to accomplish.

Key words. mathematical modeling, modeling weeks, poroelasticity, nonlinear diffusion equation

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1. Introduction and Motivation. What's the best way to teach mathematical modeling? This question has exercised the minds of many involved in both undergraduate and postgraduate programs. Even when we've agreed on what is meant by "mathematical modeling" (which in itself is by no means easy), we still have to decide why and how we are going to attempt to teach it to students. In this short study we will assume that our reason for wanting to teach modeling to students is that we want them eventually to ply their trade in industry. There they will meet a variety of different problems. The constantly evolving nature of most industries means that the majority of the processes that they will encounter are unlikely to be in their personal area of expertise. Flexible and adaptable thinking and the ability to improvise will be necessary; so will team work and communication skills. Given the typically solitary nature of many mathematical investigations, how are we to teach all of these key skills in a framework that is mathematical enough to satisfy the traditional requirements of academic assessment procedures?

One possible answer lies in the concept of "modeling weeks." In the last few decades activities of this sort have been regularly taking place in Europe (led by ECMI, the European Consortium of Mathematics in Industry), Canada and the United States (led by PIMS, the IMA, and SIAM), Latin America (led by SMM, UNAM, and ITAM), and most recently in India at IIT Bombay. The formula is simple but effective. On Monday, a real industrial problem is presented to a group of students. Four to ten students per group seems to be ideal, and of course there can be many groups and many different problems. On Tuesday, Wednesday, and Thursday, each group works

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†School of Mathematics, University of Southampton, Southampton SO17 1BJ, UK (adf@maths.soton.ac.uk, cpp@maths.soton.ac.uk).

nonstop on its problem, usually under the guidance of an experienced academic. On Friday they present their results, and at a later date the group works together (if necessary, via e-mail and the WWW) to produce a full technical report.

A modeling week can benefit the participants in a multitude of ways. New mathematical techniques and approaches, teamwork skills, independent learning skills, technical presentation and communication skills, computing experience, and experience in the art of conducting an efficient literature search can all be developed. The main components that are required to make the whole business work are (i) a group of willing and mathematically gifted students, (ii) a separate room for each group, (iii) a group leader who is helpful but who nevertheless lets the students "go their own way" as much as possible, and (iv) a good problem. This last requirement is crucial. To learn real modeling and develop the ability to "go it alone" in industry, it is essential that real *modeling* should be required, rather than simply the ability to run pre-prepared codes or solve standard textbook problems.

One of us (ADF) has been running modeling weeks for a number of years in Mexico. Normally, I provide the problems for the students to work on. In 1999 the Mexican modeling weeks were discussed at a SIAM regional meeting in Claremont, CA. During this discussion, Prof. Jorge Velasco (from UAM-Iztapalapa in Mexico City) pointed out that while it was all very well solving interesting industrial problems from European industry, Mexican students might find it a great deal more stimulating if at least some of the problems came from Latin American industry. Challenged to provide such a problem, Velasco instantly came up with the problem of automating the separation of coconuts, as described in section 2. The remainder of this study attempts to show how this interesting problem could be considered (with varying degrees of modeling and mathematical sophistication) at a typical modeling week.

2. Coconut Separation. Everybody is familiar with coconuts. (The dimensions of a typical coconut are shown in Figure 2.1.) Most of us know them as a source of refreshing coconut milk and crunchy, nutty "white." Coconuts, however, have many more uses than as an occasional snack or slightly exotic cooking component. Coco peat is used for growing orchids in Sri Lanka, and coir is used in landscape architecture and erosion control. Woven coir fibres are used as floor coverings, and sculptured husks have been used as drinking vessels since prehistoric times. In equatorial countries with high population densities, coconut husk is an important source of fuel and coconut meat is eaten in large quantities. More recently, coconuts have found other uses: carbonized husk is an extremely efficient filter material and is used in the latest generation of protective masks to combat civil and military airborne toxin threats. Even the sap has attracted novel uses, for its light-fast properties make it an ideal base for a paint (the artist Leo Villaflor from Leyte in the Phillipines has become famous for his coconut sap art). Finally, we cannot help but mention that for nearly 200 years coir matting has provided indubitably the best surface on which to play cricket if no grass is available.

All of the uses of coconuts described above require the meat to be separated from the shell and husk. One simple way of doing this is by using a combination of a machete and brute force. The coconut is chopped up, and, with skill, most of the white can be separated from most of the shell with only a few hacks. Unfortunately, this is (literally) a hit-and-miss process and is labor-intensive and low-tech. Moreover, anybody who has tried to extract the meat from a coconut will know that it is nontrivial to separate the meat from the shell, as it appears to be "glued" on. Using only a machete, perfect separation is thus well-nigh impossible.

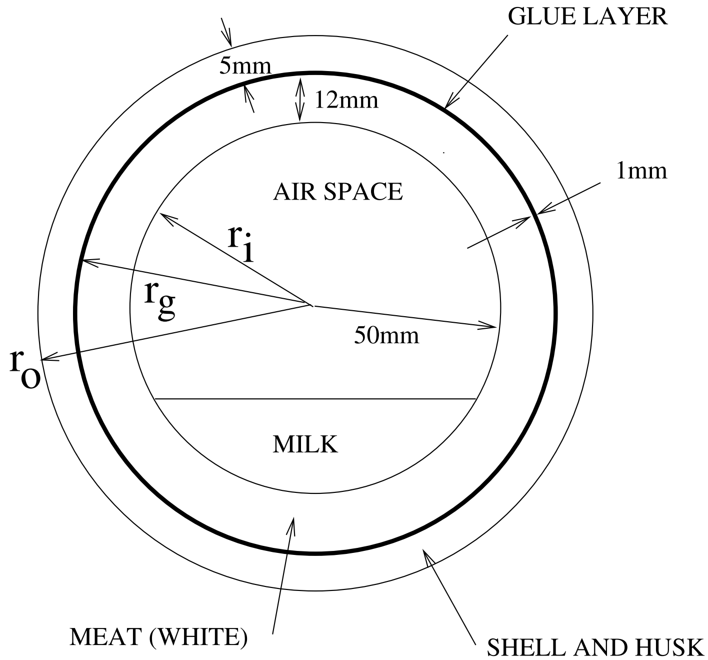

Fig. 2.1 *Schematic diagram and dimensions of a typical coconut.*

How can we automate the separation process and make the whole business more accurate and efficient? The following suggestion (based on a few fairly informal experiments) has been made:

> *Before separation, place the coconut inside a pressure vessel and apply a given overpressure for a given time. When the coconut is removed from the pressure vessel and chopped in half with a machete, the meat will fall away cleanly from the shell and no further cuts will be required.*

Here, then, is the problem: Can we propose a mathematical model for this suggested separation method that will give us some hope of determining (i) whether the process works, (ii) how much pressure should be applied, and (iii) how long it needs to be applied for?

3. Mathematical Modeling. What's the best way to start modeling the problem? For a slightly nervous group who do not know each other, the first steps can be the hardest. Often, it is best to start with a general discussion of the physical problem. However, students new to modeling are usually keen to attack a well-defined problem, as their mathematical training has primarily been a method-oriented one. In this case it is probably best to slowly direct the students toward the problems of finding the stresses and deformations that exist within the coconut when the pressure is applied. One cleanly defined mathematical problem that we can readily solve consists of examining the deformation of a linearly elastic spherical annulus under imposed pressures. This problem is worth solving, as it will be useful later and can also subsequently be used to motivate further discussion of modeling assumptions and improvements.

Let us assume that the internal and external radii of the annulus and imposed pressures are r_i and $r_o(> r_i)$ and p_i and $p_o(> p_i)$, respectively. Then, denoting the Lamé constants by λ and μ, respectively, we have to solve the equilibrium equations

$$\nabla \cdot T = 0$$

(some discussion may take place concerning why there are no time derivatives), where the stress tensor T is given by

$$T_{ij} = \lambda E_{kk}\delta_{ij} + 2\mu E_{ij}$$

and E is the infinitesimal strain tensor defined in terms of the displacements $\boldsymbol{U}$ by

$$E_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right).$$

The obvious choice of coordinates to use is spherical polars (r, θ, ϕ) in which the equilibrium equations become (see, for example, [2])

$$(T_{rr})_r + \frac{1}{r}(T_{r\theta})_\theta + \frac{1}{r\sin\theta}(T_{r\phi})_\phi + \frac{1}{r}(2T_{rr} - T_{\theta\theta} - T_{\phi\phi} + \cot\theta T_{r\theta}) = 0, \tag{3.1}$$

$$(T_{r\theta})_r + \frac{1}{r}(T_{\theta\theta})_\theta + \frac{1}{r\sin\theta}(T_{\theta\phi})_\phi + \frac{1}{r}(\cot\theta(T_{\theta\theta} - T_{\phi\phi}) + 3T_{r\theta}) = 0, \tag{3.2}$$

$$(T_{r\phi})_r + \frac{1}{r}(T_{\theta\phi})_\theta + \frac{1}{r\sin\theta}(T_{\phi\phi})_\phi + \frac{1}{r}(3T_{r\phi} + 2\cot\theta T_{\theta\phi}) = 0. \tag{3.3}$$

Let us now assume that for the simple pressure expansion problem described above, the displacement takes the form $U = U(r)\boldsymbol{e}_r$, where $\boldsymbol{e}_r$ is a unit vector in the r-direction. In this case the stresses reduce to

$$T_{rr} = (\lambda + 2\mu)U_r + \frac{2\lambda U}{r}, \qquad T_{\theta\theta} = T_{\phi\phi} = \lambda U_r + \frac{2(\lambda+\mu)U}{r}, \tag{3.4}$$

and

$$T_{r\theta} = T_{r\phi} = T_{\theta\phi} = 0.$$

Only the r-equilibrium equation survives, yielding

$$(T_{rr})_r + \frac{1}{r}\left(2T_{rr} - T_{\theta\theta} - T_{\phi\phi}\right) = 0.$$

We quickly find that

$$\left(U_r + \frac{2U}{r}\right)_r = 0$$

and thus

$$U = \frac{Ar}{3} + \frac{B}{r^2},$$

where A and B are arbitrary constants. The boundary conditions are that $T_{rr} = -p_i$ on $r = r_i$ and $T_{rr} = -p_o$ on $r = r_o$ (concerns about the signs of the pressures are likely to cause intense discussion in the group!), and thus

$$U = \frac{1}{r_o^3 - r_i^3}\left[\frac{r(p_i r_i^3 - p_o r_o^3)}{(3\lambda + 2\mu)} - \frac{(p_o - p_i)r_o^3 r_i^3}{4\mu r^2}\right]. \tag{3.5}$$

Table 3.1 *Elastic displacements at the inside ($r = r_i = 50$ mm) and outside ($r = r_o = 68$ mm) of a hollow sphere for a 3 atm external pressurization for various different materials.*

Material	λ (Pa)	μ (Pa)	U_i (m)	U_o (m)
India rubber	2.7×10^9	1.7×10^8	-2.7×10^{-5}	-1.7×10^{-5}
Steel	1.1×10^{11}	8.2×10^{10}	-9.4×10^{-8}	-8.7×10^{-8}
Glass	3.7×10^{10}	3.2×10^{10}	-2.5×10^{-7}	-2.4×10^{-7}
Walnut (radial)	1.1×10^{10}	4.4×10^9	-1.5×10^{-6}	-1.2×10^{-6}

There is a little more to (3.5) than first meets the eye. The predicted displacement depends on both pressures and not just on the pressure difference, a fact that the students might find counterintuitive. Moreover, though when $p_i < p_o$ it is clear that both surfaces of the annulus experience a negative displacement, if the pressures are such that $p_i > p_o$, then the surface displacements may be either positive or negative. Before continuing, it is also instructive to see roughly how large we may expect typical elastic displacements to be. Interestingly, many students will be completely stymied by the difficulty of finding the elastic constants for coconut. The importance of the technique of seeing how the numbers turn out for a range of similar materials soon becomes clear, for when we take $r_i = 50$ mm, $r_o = 68$ mm, $p_i = 1$ atm, and $p_o = 3$ atm, we find that for various different materials (whose Lamé constants are easily found; see, for example, [3] and [4]) the respective inner and outer displacements U_i and U_o are given in Table 3.1; the corresponding stresses at the "glue" layer ($r = r_g = 62$ mm) are given by $T_{rr}(r_g) = -2.6 \times 10^5$ Pa and $T_{\theta\theta}(r_g) = T_{\phi\phi}(r_g) = -5.2 \times 10^5$ Pa.

We conclude that, as might be expected, a pressure difference of only two atmospheres produces rather small displacements.

3.1. The Separation Mechanism. Armed with the simple solution (3.5), let us now try to consider how the separation mechanism might work. When the pressure is applied, how can elastic effects somehow cause a displacement of the meat away from the shell? One possible route forward here is for the students to actually carry out a two-layer elastic calculation, imposing continuity of both normal stress and displacement at the shell/meat interface (the tangential stress is zero). Such a calculation will be equivalent to that performed in section 3.3 with $\psi = 0$. One observation that the students may make is that the stress is everywhere compressive, suggesting that no separation can occur.

A similar effect can be observed for the one-layer problem by noting that, from (3.4), we have

$$T_{rr} = \frac{-r_o^3 p_o (r^3 - r_i^3) - r_i^3 p_i (r_o^3 - r^3)}{r^3 (r_o^3 - r_i^3)} \tag{3.6}$$

so that the radial stress T_{rr} is always negative. (Some interesting discussions may be sparked off here by asking the group whether they are surprised that the radial stress seems to be independent of the elastic constants.) How can the meat be forced away from the shell? Students in a group are often tempted to rely only on their mathematical abilities and frequently have great difficulty proposing *mechanisms*, but it is now the mechanism that has become the key issue. The mathematics tells us that we must impose two conditions at the interface. These two conditions must therefore reflect the separation mechanism. Discussion should highlight issues such as the strength of the glue at the interface, the fact that, until separation occurs, the two materials are strongly bonded together, and the fact that the stresses must be tensile rather than compressive to break the glue. This suggests that any purely elastic model in which we impose a pressure on the outside of the sphere that is higher than the pressure on the inside will invariably lead to stresses that are everywhere compressive (i.e., negative, as in (3.6)) in the radial direction. The glue can therefore never fail.

One way that the process *could* work is if the pressure difference were to be applied across the meat alone. The load would then be carried by compression of the meat and inward movement of the coconut shell. If there were no pressure difference across the shell, then this movement could be generated by imposing a tensile stress at the meat/glue interface. It is this tensile (positive) stress that might cause the glue to fail. After some further debate, it should emerge that the most likely way that the desired result can be achieved is therefore if the shell of the coconut is porous, but the interface between the meat and the shell and/or the meat itself is impermeable.

Now that we have a mechanism, can we (a) satisfy ourselves that the required conditions really do pertain, and (b) model the process and make some realistic predictions?

3.2. Is a Coconut Porous? None of the calculations that have been carried out thus far would be absent from a standard elasticity textbook. It is clear, however, that our new proposed mechanism for separation will require some altogether more sophisticated modeling. It is worth pointing out that it is nearly always the case that an ideal modeling week problem includes both a "standard" element (so that the group can get started and produce some concrete results in a short space of time) and a "research" element, where a partial rethink and some genuinely new modeling input are required.

We have proposed a separation mechanism that relies on the fact that the shell of a coconut is porous, and this will be investigated further below. First, however, we ought to check whether or not such porosity is realistic. Unfortunately, there appears to be little information in the literature (or even on the Internet) that might tell us whether a coconut shell is porous and, if it is, how porous.

Modeling weeks and industrial study groups have a proud tradition of "informal" experimentation. In the absence of the required literature, we therefore decided to perform our own experiments. The procedure was simple. We purchased a coconut and sawed it in half. (The cut was made along the equator with the "eyes" of the nut positioned at the North Pole.) We removed the meat from one half of the coconut but not from the other and placed both halves of the coconut in the tops of two measuring jugs (a typical coconut fits nicely into the top of a typical jug). We then filled both hemispherical shapes with water and waited. The results were unequivocal and surprisingly repeatable: no water ever passed through the coconut half where the meat remained, but the half where the meat had been removed leaked water at a rate of about 20 ml in 4 days.

Our experimental results now allow us to make a rough estimate of the relative permeability of a coconut shell. Porous media satisfy Darcy's law (see, for example, [1]), which states that the flow velocity (specific discharge) $\boldsymbol{q}$ of flow through a porous medium under gravity and an applied pressure field p is given by

$$\boldsymbol{q} = -\frac{k}{\mu}\nabla(p + \rho g z), \tag{3.7}$$

where k is the relative permeability of the porous medium (measured in units of m^2) and ρ and μ are, respectively, the density and dynamic viscosity of the fluid. In our experiment we rely solely on gravity to produce a flow. Thus, rearranging (3.7), we find that $\boldsymbol{q} = (0, 0, w)$ and

$$k = -\frac{\mu w}{\rho g}.$$

Let us now assume that the coconut shell is hemispherical (this is not very far from the truth). The measured distance to the outer surface of the shell was 68 mm, so the surface area of porous material was $2\pi \times (68 \times 10^{-3})^2$ m^2. Since this surface area gave a flow of 20 ml in 4 days, we may estimate w to be

$$w \sim -\frac{20 \text{ ml}}{(4 \text{ days}) \times 2\pi \times (68 \times 10^{-3})^2 \text{ m}^2} \sim 2 \times 10^{-9} \text{ m/s}.$$

Now using $\mu/\rho = 10^{-6}$ m^2/s and $g = 9.8$ m/s^2 gives

$$k \sim 2 \times 10^{-16} \text{ m}^2. \tag{3.8}$$

Of course, we must be modest in our claims for (3.8). The experiments, though repeatable, are (literally) of the "kitchen sink" variety. Compromises have also been made in calculating the gravitational force on the hemisphere of water contained by the shell. All of these calculations may be performed with greater accuracy if desired; we do not do so as we are interested only in an order-of-magnitude estimate. As a reality check, however, we note that since a typical value for radial permeability in spruce heart wood is given by [6] as 5×10^{-17} m^2, it seems at least plausible that the order of magnitude of (3.8) is correct.

Given that we now know that a coconut is porous and we have at least a rough estimate for its relative permeability, does this give us any information about how long the pressure should be applied for? Evidently time is required for the outer pressure to "reach" the glue layer. We can estimate how long this might take by noting that the equation of mass conservation for the air in the coconut pores is

$$(\rho\psi)_t + \nabla.(\rho\psi\boldsymbol{v}) = 0, \tag{3.9}$$

where ρ is the air density, ψ is the porosity (i.e., the fraction of the cross-sectional area that is occupied by pores) of the coconut shell, and $\boldsymbol{v}$ is the velocity of the air *in the individual pores*. To relate $\boldsymbol{v}$ to the specific discharge $\boldsymbol{q}$ we make the simple assumption that

$$\boldsymbol{v} = \boldsymbol{q}/\psi. \tag{3.10}$$

Henceforth we shall also assume that ψ is constant. This is a simplistic model, and for wood-like materials in particular a tensor formulation of (3.10) is required to take account of the effects of grain. It is also quite difficult to measure ψ accurately. Many sources such as [5] give values for porosity for a wide range of materials such as sands and rocks, and all of these lie between 0.1 and 0.15, but wood is surprisingly porous: [6] gives the porosity of spruce heart wood as 0.733 and [7] cites values of between 0.6 and 0.7.

We will also assume for simplicity that the shell is "thin" compared to the radius of the coconut so that we may work in a local coordinate x normal to the shell and with a local specific discharge $\boldsymbol{q} = u\boldsymbol{e}_x$, where $\boldsymbol{e}_x$ is a unit vector in the x-direction.

Ignoring the effects of gravity (the group should be encouraged to justify this!), (3.7) now gives

$$u = -\frac{k}{\mu} p_x. \tag{3.11}$$

We now use the perfect gas law

$$p = \rho R T, \tag{3.12}$$

where R is the universal gas constant and T is the temperature. As usual for a porous medium flow, we assume that T is constant, since we expect the gas to quickly equilibrate with the temperature of the shell. Elimination of ρ and u between (3.9), (3.10), (3.11), and (3.12) now yields

$$p_t = \left(\frac{k}{\mu\psi}\right)(pp_x)_x, \tag{3.13}$$

which is a nonlinear diffusion equation for the pressure p. Rather than embarking immediately on a search for similarity solutions or numerical schemes for (3.13), we note that a time scale τ for the transmission of the pressure increase through the shell may easily be derived from (3.13). We find that

$$\tau = \frac{\mu\psi L^2}{kP}, \tag{3.14}$$

where L is a typical x-length and P a typical overpressure. Many mathematics students seem unwilling to derive quick estimates such as (3.14), but even such simple formulae can tell us a great deal. For example, the transmission time increases as L and μ increase and decreases as P and k increase, as expected. Some of the students may initially be surprised that the transmission time increases with increasing ψ, but for fixed k increasing ψ means that there is more gas to compress. In reality, k is normally a strong function of ψ. We can now use our experimental results: with $L \sim 5$ mm, $\mu \sim 2 \times 10^{-5}$ Ns/m^2, and $P \sim 1$ atm we find that, with $k = 2 \times 10^{-16}$ m^2,

$$\tau \sim 25\psi\ s. \tag{3.15}$$

Although the timescale estimate (3.15) involves many approximations, it tells us (i) that the separation process will not be instantaneous, and (ii) that it is nevertheless likely that the mechanism will be quick enough (for typical values of ψ mentioned above, possibly around 10 seconds) to be a practical proposition.

3.3. A More Advanced Model. We are now ready to pose a more advanced model for coconut separation by means of pressure application. The key to modeling the whole process is to understand that the porous medium flow in the shell must be coupled to the elastic problem. Although the pressure profile is not influenced by the elastic displacements that occur within the coconut, the changes in pressure provide a body force that contributes to the elastic equilibrium.

To propose a coupled model of coconut separation, we distinguish between three different regions: the space inside the coconut ($r < r_i$), the "meat" of the coconut ($r_i < r < r_g$), and the "shell" region ($r_g < r < r_o$). We assume that the thin "glue" layer that separates the shell from the meat has zero thickness and is situated at

$r = r_g$; as discussed above, we shall also assume that the shell is a homogeneous isotropic elastic porous medium of relative permeability k with Lamé constants λ_s and μ_s, while the meat is an impermeable homogeneous isotropic elastic body with Lamé constants λ_m and μ_m. The following problems thus apply in each region:

Air space ($0 < r < r_i$):

$$p = p_i, \qquad \text{no elasticity.}$$

Meat ($r_i < r < r_g$):

$$(T^m_{rr})_r + \frac{1}{r}(2T^m_{rr} - T^m_{\theta\theta} - T^m_{\phi\phi}) = 0, \qquad \text{no air.}$$

Shell ($r_g < r < r_o$):

$$(T^s_{rr})_r + \frac{1}{r}(2T^s_{rr} - T^s_{\theta\theta} - T^s_{\phi\phi}) = \psi p_r, \qquad p_t = \left(\frac{k}{\mu\psi}\right)\frac{1}{r^2}(r^2 p p_r)_r.$$

Here T^i (with $i = m$ or s) is given in terms of the displacements U^i by (3.4). The boundary conditions must now be discussed. At time $t = 0$ we have $p = p_i$ in the shell and the air space. For $t > 0$ we have $p = p_o$ at $r = r_o$ (we assume that the pressure vessel is "instantaneously" pressurized), $p_r = 0$ at the impermeable "glue" layer, and $r = r_g$ and $p = p_i$ in the air space. As far as the boundary conditions for the elasticity problem are concerned, we have $T^s_{rr} = -p_o$ at $r = r_o$ and $T^m_{rr} = -p_i$ at $r = r_i$. The conditions at the glue layer take a little more thinking about. The displacement must be continuous across the glue layer, so that

$$[U]^{r_{g+}}_{r_{g-}} = 0.$$

Finally, continuity of normal force dictates that we must have

$$T^m_{rr}\,|_{r=r_{g-}} = T^s_{rr}\,|_{r=r_{g+}} - \psi p(r_g, t);$$

if the stress in the shell at the glue interface is sufficient to overcome the bonding strength of the glue, then the shell will detach from the meat and the process will have worked successfully.

Now that we have a detailed model for the process, what can we do with it? This will depend on the expertise of the individual group members. Some possibilities include:

(i) Prove existence and uniqueness of the solution to the governing equations.
(ii) Analyse the equations to determine the important quantities and scales.
(iii) Write a numerical scheme to compute solutions.
(iv) Use a commercial code to compute solutions.
(v) Try to determine the general properties and qualitative behavior of the solution to the governing equations.

Though (i) is of much mathematical interest, it is hardly likely to have the major players of the coconut industry leaping from their desks with excitement. We have already dealt with (ii) to a large extent and (iii) is relatively simple if the group contains members with experience in solving nonlinear parabolic problems numerically. (iv) can be instructive provided that a suitable code and a suitably trained user are both available; otherwise it may take a good deal longer than (iii). For the purposes of this study (and in the general spirit of mathematical modeling), let us see what progress may be made with (v).

First, we note that the problem for the pressure decouples from the elasticity problem. Essentially, we have to solve

$$p_t = \frac{\alpha}{r^2}(r^2 p p_r)_r \qquad \left(\alpha = \frac{k}{\mu\psi}\right) \tag{3.16}$$

with

$$p_r(r_g, t) = 0, \qquad p(r_o, t) = p_o, \qquad p(r, 0) = p_i.$$

Though this problem is nonlinear and cannot be solved in closed form, it is not hard to predict the behavior of p for small times and large pressure changes. Problem (3.16) has similarity solutions of the form

$$p = t^k f(\eta) \quad (\eta = r^a/t)$$

with $a = 2/(1+k)$ and

$$4\eta\alpha(ff'' + f'^2) + ((k+1)^2\eta^{k+1} + 2(k+3)\alpha f)f' - k(k+1)^2\eta^k f = 0,$$

and the group should be encouraged to examine various asymptotic limits by nondimensionalizing and setting $p = (p_o - p_i)\bar{p}$, $r = r_g + (r_o - r_g)\bar{r}$, and $t = (r_o - r_g)^2/(\alpha(p_o - p_i))$ so as to identify the nondimensional parameters $(r_o - r_g)/r_o$ and $p_i/(p_o - p_i)$. A host of other cases (large time solutions, small pressure differences, etc.) may also be examined. None of this makes much difference to the fact, however, that the elevated value of p_o diffuses into the shell, and after a reasonable time the pressure becomes close to p_o everywhere in the shell; numerical calculations are required only to determine the exact details of how this happens.

As far as the elasticity problem is concerned, we note that this may (at least in principle) be solved in closed form. We find that

$$U^m = \frac{C_1 + C_2 r^3}{r^2}, \quad U^s = \frac{1}{r^2}\left(C_3 + \int_{r_g}^{r} \beta^2 \left(C_4 + \frac{\psi(p(\beta,t) - p(r_g,t))}{\lambda_s + 2\mu_s}\right) d\beta\right),$$

where C_1, C_2, C_3, and C_4 are constants that may be determined once $p(r,t)$ is known.

One obvious special case immediately attracts our attention: suppose that the pressurization process is nearly complete, so that essentially $p(r,t) = p_o$ everywhere. In this case we find that, as in section 3,

$$U^m = \frac{C_1 + C_2 r^3}{r^2}, \quad U^s = \frac{C_3 + C_4 r^3}{r^2},$$

and the constants may be determined immediately (though with some pain: using Maple or Mathematica is advisable). As far as the debonding of the glue is concerned, the key quantity is the normal stress at $r = r_{g+}$. We find after some work that

$$T^s_{rr}(r_g) = \frac{\alpha_1}{\beta_1},$$

where

$$\alpha_1 = 3p_i r_i^3 \mu_s(3\lambda_s+2\mu_s)(\lambda_m+2\mu_m)(r_o^3-r_g^3)+3p_o r_o^3\mu_m(r_g^3-r_i^3)(\lambda_s+2\mu_s)(3\lambda_m+2\mu_m)$$
$$-p_o\mu_s\psi(r_o^3-r_g^3)(3\lambda_s+2\mu_s)(3\lambda_m r_i^3+4\mu_m r_g^3+2\mu_m r_i^3)$$

and

$$\beta_1 = -(r_g^3-r_i^3)(3\lambda_m+2\mu_m)\mu_m(3r_o^3\lambda_s+2\mu_s(r_o^3+2r_g^3))$$
$$-(r_o^3-r_g^3)(4r_g^3\mu_m+3\lambda_m r_i^3+2\mu_m r_i^3)\mu_s(3\lambda_s+2\mu_s).$$

From this it is easy to show that the tensile radial stress that we desire ($T_{rr}(r_g)>0$) will occur if

$$\psi(4\mu_m r_g^3+(3\lambda_m+2\mu_m)r_i^3) > \frac{3r_i^3 p_i}{p_o}(\lambda_m+2\mu_m)$$
$$+\left(\frac{3r_o^3(r_g^3-r_i^3)}{(r_o^3-r_g^3)}\right)\left(\frac{\mu_m(\lambda_s+2\mu_s)(3\lambda_m+2\mu_m)}{\mu_s(3\lambda_s+2\mu_s)}\right), \tag{3.17}$$

a result that has an interesting dependence on the dimensions of the coconut and the elastic constants and also involves only the pressure ratio.

Various calculations may be made using (3.17), but the students' first thought will probably be to use some realistic-looking guesses to see whether or not a tensile stress can be produced. Using values of $r_i=50$ mm, $r_g=62$ mm, $r_o=68$ mm, $p_i=1$ atm, $p_o=3$ atm, and $\psi=0{\cdot}6$, we find that for elastic constants $\lambda_s=1.1\times10^{10}$ Pa, $\mu_s=4.4\times10^9$ Pa (values for pine), $\lambda_m=2.7\times10^9$ Pa, and $\mu_m=1.7\times10^8$ Pa (values for rubber),

$$T^s_{rr}(r_g)\sim 3\times10^4 \text{ Pa},$$

a tensile stress of over one-quarter of an atmosphere. Of course, many more calculations must be carried out (for example, with these parameters we find an external pressure of 5 atm is required to produce a tensile stress at the glue of 1 atm) and much work remains for the group to do to understand exactly how the tensile stresses depend on the physical parameters of the problem. But the main point is that the principle has been established and the separation mechanism works. At this point the problem may be considered to officially be "a success."

4. Conclusions and Further Work. The "modeling week" modus operandi has been used for long enough now for us to know that its track record is impressive. This method of learning mathematics is fun and rewarding as well as being both technically and politically correct. However, as we noted above, good team leaders are desirable and good problems are essential. We believe that the coconut separation problem described above forms a useful role model of a "good" problem because

- it is a real problem where results may be of real use,
- it is easy to understand,
- the modeling must be based on both mathematical theory and physical intuition,
- it is possible to perform some simple yet valuable calculations to "get started,"
- it can be attacked using many different levels of sophistication, and
- it is amenable to theoretical, numerical, and even experimental techniques.

One of the key components of the "modeling week philosophy" is that the final presentation and report should not just say, "We did some mathematics; here it is," but should try to give concrete answers to the problem(s) originally posed. In this instance our model has given us a good idea of how much pressure will be required, how long the process will take, and what experiments should be carried out if we wish to seriously consider the process as a going industrial concern.

Of course, many modeling refinements are possible. Will the external pressure really be imposed as a step change? Does the pressure deform the porous medium, rendering ψ nonconstant? Is the meat layer really impermeable? Is the whole process isothermal? What form does the "glue" really take and how does the process of debonding occur? Is the slight asymmetry of a typical coconut important? These and many more questions can be posed and discussed by the group. By this stage, however, it will probably be late on Thursday afternoon and most of the group will have departed to write their full-color, animated PowerPoint presentation for tomorrow. . . .

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